# International Trade Fluctuations and Monetary Policy*

Fernando Leibovici
York University

Ana Maria Santacreu
St. Louis Fed

September 2015

**Abstract**

This paper studies the role of trade openness for the design of monetary policy. We extend a standard small open economy model of monetary policy to capture cyclical fluctuations of international trade flows, and parametrize it to match key features of the data. We find that accounting for trade fluctuations matters for monetary policy: when the monetary authority follows a Taylor rule, inflation and the output gap are more volatile. Moreover, we find that the volatility of these variables is significantly higher when the central bank follows the optimal policy based on a model that cannot account for international trade fluctuations.

*We thank seminar participants at York University, the Midwest Macroeconomics meetings (Miami, FL), the NYU Alumni Conference, and the European Economic Association meetings (Mannheim, Germany) for helpful comments. We also thank Usa Kerdnunvong for excellent research assistance. The views expressed are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors. Contact information: flei@yorku.ca, am.santacreu@gmail.com.

# 1 Introduction

In recent decades, academics and policymakers have paid increasing attention to the role of trade openness on the design of monetary policy. Open economies are exposed to additional sources of shocks arising, for instance, from changes in exchange rates or foreign demand. Recent papers, therefore, investigate how monetary authorities should respond to economic fluctuations in this context (see, for instance, Gali and Monacelli (2005), De Paoli (2009), Lombardo and Ravenna (2014)). These papers, however, typically rely on open economy models that cannot account for the dynamics of international trade flows at business cycle frequencies. In this paper, we evaluate the importance of trade openness for monetary policy in an environment that accounts for salient features of international trade fluctuations.

One common feature of these models is a constant elasticity of substitution (CES) demand system. While this assumption implies a unitary income elasticity of imports, previous studies have estimated it to be higher in the data. Similarly, while these models are typically calibrated to feature an import price elasticity that is higher than one, this elasticity is estimated to be well below one in the data (Marquez (2013), Leibovici and Waugh (2014)). This failure to capture a key transmission channel of foreign shocks puts into question the validity of previous findings on the role of trade openness on the design of monetary policy.[1]

We build upon a standard small open economy model, following Gali and Monacelli (2005), and extend it by introducing a time-varying trade wedge, whose functional form is designed to match the empirical income and price elasticities of imports that we estimate in the data. This approach is motivated by recent papers which show that deviations of imports between standard models and the data are systematic and operate as a time-varying trade wedge in the demand for foreign goods (Levchenko, Lewis, and Tesar (2010), Alessandria, Kaboski, and Midrigan (2013), Leibovici and Waugh (2014)). Therefore, our modeling strategy allows us to capture alternative mechanisms that may account for fluctuations of international trade flows (Alessandria, Kaboski, and Midrigan (2010), Chor and Manova (2012), Eaton, Kortum, Neiman, and Romalis (2011), among others), while remaining agnostic about the specific mechanisms at play.

The model consists of a small open economy populated by a representative household, who trades a complete set of Arrow securities with the rest of the world and supply labor endogenously. A continuum of monopolistically competitive firms use labor to produce varieties and sell them to domestic and foreign consumers, with prices sticky à la Calvo

[1] A number of recent studies have, more generally, documented the failure of standard models to account for salient features of international trade fluctuations along a number of other dimensions. For instance, Heathcote and Perri (2002), Alessandria, Kaboski, and Midrigan (2013), and Engel and Wang (2011) show that standard international business cycle models imply that international trade flows are not as volatile and pro-cyclical as in the data. Similarly, a number of papers have documented that standard models cannot account for the collapse of international trade during the recent crisis (Levchenko, Lewis, and Tesar (2010), Eaton, Kortum, Neiman, and Romalis (2011), Alessandria, Kaboski, and Midrigan (2010), and many others).

(1983). The central bank conducts monetary policy following a Taylor rule through which it adjusts the nominal interest rate in response to changes in the previous period's nominal interest rate, current inflation, and output. Finally, aggregate fluctuations are driven by shocks to aggregate domestic productivity as well as by shocks to foreign demand.

Our main departure from the standard model consists of a time-varying trade wedge that shifts consumers' preferences between domestic and imported goods. We model the trade wedge to be such that the implied income and price elasticities are a simple function of parameters. In the empirical section of the paper, we estimate the price and income elasticities in the data for a number of small open economies (Australia, Canada, New Zealand, and United Kingdom). Following Gali and Monacelli (2005), we calibrate our economy to Canada, and choose the trade wedge parameters to match these elasticities.

We first examine the implications of the model for features of international trade fluctuations not targeted directly in our calibration. To do so, we contrast the business cycle implications of our model with those implied by its counterpart with a constant trade wedge, while keeping the Taylor rule constant across the two models. To simplify the analysis, we assume that the monetary authority follows a simple Taylor rule which is only a function of current inflation. Our extension of the model improves its fit of the data along a number of dimensions. In contrast to its standard countepart with a constant trade wedge, our model can account for the high volatility of imports and exports relative to GDP, the counter-cyclicality of net exports, the pro-cyclicality of imports, and the positive correlation between imports and exports.

We then use this economic environment to study the impact of the trade wedge on the volatility of economic variables that may be relevant for monetary policy. We find that our model generates higher inflation and output gap volatility relative to the standard model. These results suggest that accounting for international trade fluctuations is important for the design of monetary policy.

To examine the extent to which this is the case, we solve the constrained Ramsey problem, where the planner is constrained to choose the parameter values of the Taylor rule. Specifically, we compute the Taylor rule coefficients that maximize the lifetime expected utility of the representative consumer in a competitive equilibrium. We find that the monetary authority should react differently to changes in inflation and output under each model. In particular, in a model that can better account for trade fluctuations, the optimal monetary policy implies a faster speed of nominal interest rate adjustment and a stronger response to inflation and output fluctuations.

Finally, we evaluate whether differences in the optimal policy parameters matter for business cycle dynamics. To do so, we assume that the true data generating process is the time-varying trade wedge model, but the central bank designs policy based on the optimal Taylor rule of the constant trade wedge model. We find that conducting monetary policy based on the misspecified model would almost double the volatility of inflation while also increasing the volatility of the output gap fluctuations.

By showing that trade fluctuations matter for the design of monetary policy, these results put in context previous findings in the literature. In particular, in a model that better accounts for international trade fluctuations, without significantly affecting other business cycle dynamics, we find that central banks should conduct monetary policy differently than implied by standard models. Our paper, thus, introduces recent developments from the literature on international trade dynamics to the established literature that studies monetary policy in open economies.

The rest of the paper is organized as follows. Section 2 introduces a general log-linear specification of imports demand and uses it to contrast the implications of standard models with the data. Section 3 presents the model. Section 4 specifies the time-varying trade wedge and derives its implications for the trade elasticities. Section 5 calibrates the model and analyzes its quantitative implications. Section 6 studies the optimal design of monetary policy. Section 7 concludes.

# 2 International trade fluctuations: theory vs evidence

In this section, we document salient features of international trade fluctuations and contrast them with the implications of standard models of international trade. Our approach follows previous work in the literature that uses the demand for imports to characterize international trade flows as a function of economic activity and prices.[2] To do so, we specify a log-linear imports demand equation which nests a large class of models of international trade. We begin by examining these model's implications for imports demand, and contrast these implications with estimates for several small open economies.

## 2.1 Log-linear demand for imports

We begin this section by specifying a log-linear imports demand equation to contrast the implications of standard models of international trade with the data:

$$\log M_t = \kappa + \varphi \log \frac{p_{m,t}}{P_t} + \chi \log A_t + \nu_t \tag{1}$$

where $M_t$ denotes real imports, $\kappa$ is a constant, $\frac{p_{m,t}}{P_t}$ denotes the price of imports relative to an absorption price index, $A_t$ denotes real absorption, and $\nu_t$ is an error term that captures

[2] Most closely related to our approach are Leibovici and Waugh (2014) and Levchenko, Lewis, and Tesar (2010); these papers build on the influential work of Houthakker and Magee (1969) and Feenstra (1994).

deviations of imports not explained by the first two terms.[3]

We refer to $\varphi$ and $\chi$ loosely as price and income "elasticities", respectively, while remaining agnostic about their structural nature; we simply think of them as moments of the data that characterize the statistical properties of imports, income, and prices. Moreover, note that, while the measure of economic activity that we focus on is absorption, we refer to absorption, income, and output interchangeably throughout the paper.[4]

## 2.2 Trade elasticities in standard models

Standard models of international trade have sharp implications for the trade elasticities $\varphi$ and $\chi$. Specifically, we restrict attention to models with constant elasticity of substitution (CES) preferences or production functions, such asKrugman (1980), Anderson and van Wincoop (2003), Eaton and Kortum (2002), and Melitz (2003). This class of models also includes international business cycle models such as Backus, Kehoe, and Kydland (1992) and Heathcote and Perri (2002).

These models imply that the demand for imports is given by $\log M_t = \omega - \theta \log \frac{p_{m,t}}{P_t} + \log A_t$, where $\theta$ denotes the price elasticity of imports and we refer to $\omega$ as a "trade wedge" that is a function of structural parameters such as iceberg trade costs or home-bias. Then, in these models, the price elasticity $\varphi$ is equal to $-\theta$, and the income elasticity $\chi$ is equal to one.

## 2.3 Trade elasticities in the data

We now contrast the trade elasticities implied by standard models with estimates from time series data for several small open economies. Specifically, we use data on real imports, real absorption, and the relative price of imports to estimate $\varphi$ and $\chi$ from equation (1) following a standard ordinary least squares approach. As mentioned above, while we refer to these estimates as empirical "elasticities", we interpret them as moments of the data, remaining agnostic about their structural nature.

We focus on the following small open economies which have been previously studied in the literature: Australia, Canada, New Zealand, and the United Kingdom. We obtain the data from their respective official statistical agencies, whenever available, as well as from Haver Analytics, the OECD, and Eurostats. We include in our sample as many observations as available starting from 1980. All data is seasonally adjusted and de-trended by applying a Hodrick-Prescott filter with smoothing parameter 1600.

[3]Absorption is gross domestic product plus imports minus exports; it is a measure of aggregate demand in the economy.

[4]With balanced trade, these three objects are equal to each other.

We focus on imports and absorption of goods, and their respective relative price. In doing so, we exclude services and government expenditures. This allows us to obtain measures of $M_t$ and $A_t$ that map closely to the objects featured by standard trade models.

While data on real imports is typically provided by statistical agencies (or easy to compute by adjusting nominal imports with its corresponding price index), this is not generally the case for real absorption. In the countries that we study, real measures of the components of GDP required to compute total absorption are provided as chain-type indexes of the type proposed by Fisher (1922). While desirable along some dimensions, they are not additive across categories (see Ehemann, Katz, and Moulton (2002) and Whelan (2002) for detailed discussions). This implies that real absorption cannot simply be computed by adding real GDP to real imports and substractic real exports. Therefore, we follow the approximate solution proposed by Diewert (1978), and compute real absorption as a "Fisher of Fishers" index. That is, instead of using data on quantities and prices to compute a Fisher index of absorption, we use Fisher indexes of quantities and prices for each of the categories of interest and then compute a Fisher index based on these measures.

The estimation results are presented in Table 1. The first four rows provide the estimates of the trade elasticities $\varphi$ and $\chi$ corresponding to each of the countries. While there is heterogeneity in $\varphi$ and $\chi$ across countries, two salient features emerge. First, we find that the empirical price elasticity of imports $\varphi$ is considerably below the value at which $-\theta$ is calibrated in standard trade or international business cycle models. While this parameter often takes values around -1.5 in the latter, it can take values above 4 in the former. Moreover, we find that the price elasticity $\varphi$ is very low, smaller than -0.50 in all cases (and with an average of -0.09). Second, we find that, in contrast to models with CES demand or production functions, the income elasticity of imports $\chi$ is considerably above unity in all cases. In particular, as reported in the fifth row of the table, the average income elasticity across countries is 1.47.

These findings stand in contrast with the implications of standard models, and are consistent with previous empirical estimates in the literature. In particular, following a similar approach, Leibovici and Waugh (2014) estimate price and income elasticities equal to -0.26 and 1.99, respectively, for the U.S. over the period 1967Q2 - 2013Q4.

The goal of this paper is to investigate the role of trade fluctuations for the design of monetary policy in small open economies. While a number of papers have previously investigated the drivers of the mismatch between the trade fluctuations implied by standard models and the data, we remain agnostic about its sources.

| Country | Price elasticity ($\varphi$) | Income elasticity ($\chi$) | $R^2$ | Period |
|---|---|---|---|---|
| Australia | -0.295 | 1.365 | 0.56 | 1985Q3 - 2014Q2 |
| Canada | 0.002 | 2.066 | 0.65 | 1981Q1 - 2014Q2 |
| New Zealand | -0.114 | 1.138 | 0.58 | 1987Q1 - 2013Q3 |
| United Kingdom | 0.051 | 1.318 | 0.34 | 1987Q2 - 2014Q2 |
| Average | -0.089 | 1.472 | - | - |

Table 1: International Trade Fluctuations, Import Elasticities

# 3 Model

Consider a small open economy that trades goods and financial assets with the rest of the world. The small open economy is assumed to be infinitesimal in size relative to the rest of the world; therefore, decisions in the former do not affect variables in the latter. We use subscripts $H$ and $F$ to denote variables corresponding to goods produced in the small open economy and the rest of the world, respectively. Moreover, we use superscript $*$ to denote the rest of the world's variables, and subscript $SS$ to denote variables at their deterministic steady-state level.

## 3.1 Representative household

The small open economy is populated by a representative household that maximizes lifetime expected utility, which is given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_{s,t})$$

where $\beta \in (0,1)$ denotes the subjective discount factor, $C_t$ denotes the amount of consumption of the final good, $N_{s,t}$ is the amount of labor supplied at the competitive wage rate $W_t$, and $\mathbb{E}_0$ is the expectations operator conditional on the information set in period zero. The period utility function is assumed to be given by $U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$, where $\sigma > 0$ denotes the coefficient of relative risk aversion, and $\phi > 0$ is the Frisch elasticity of labor supply.

Consumption of the final good results from aggregating domestic and foreign goods with a constant elasticity of substitution aggregator, which is given by:

$$C_t = \left[ (1-\alpha_t)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha_t^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right] \tag{2}$$

where $\eta > 0$ is the elasticity of substitution between domestic goods $C_{H,t}$ and foreign goods $C_{F,t}$, and $\alpha_t \in [0,1]$ is the time-varying trade wedge.[5] This time-varying trade wedge is our main departure from standard models, and we model it explicitly in the next section.

Domestic goods $C_{H,t}$ and foreign goods $C_{F,t}$ result, in turn, from aggregating domestic and foreign varieties with constant elasticity of substitution aggregators given by:

$$C_{H,t} = \left( \int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \qquad C_{F,t} = \left( \int_0^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $\varepsilon$ is the elasticity of substitution across domestic varieties $j \in [0,1]$.

Finally, households have access to a one-period risk-free bond at gross nominal interest rate $R_t$, as well as to a complete set of state-contingent claims through which they can insure themselves by trading with the rest of the world.

Then, the problem solved by the representative household is given by:

$$\max_{C_t, C_{H,t}, C_{H,t}(j), C_{F,t}, C_{F,t}(j), D(s^{t+1}), N_{s,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_{s,t})$$

subject to

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 P_{F,t}(j) C_{F,t}(j) dj + B_{t+1} + \sum_{s_{t+1}} \mathcal{M}(s^{t+1}|s^t) D(s^{t+1})$$

$$\leq W_t N_{s,t} + \Pi_t + R_t B_t + D_t$$

$$C_t = \left[ (1-\alpha_t)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha_t^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]$$

$$C_{H,t} = \left( \int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$C_{F,t} = \left( \int_0^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $s^t$ denotes the history of aggregate states from period 0 up to and including period $t$, $\mathcal{M}(s^{t+1}|s^t)$ is the state-$s^t$ price (in domestic currency) of an Arrow security that pays one unit of the domestic currency in state $s^{t+1}$, $D(s^{t+1})$ is the number of state-$s^{t+1}$ Arrow securities purchased, $B_{t+1}$ denotes the value of bond purchases, and $\Pi_t$ denotes the profits that accrue from the ownership of domestic firms. Note that $\mathcal{M}(s^{t+1}|s^t)$ is sometimes denoted as $\mathcal{M}_{t,t+1}$.

[5] In standard models, $\alpha_t$ determines the degree of openness and is inversely related to the degree of home-bias in preferences.

## 3.2 Firms

A unit measure of monpolistically competitive firms produce differentiated varieties $j \in [0,1]$ with a linear technology in labor represented by production function $Y_t(j) = A_t N_{d,t}(j)$, where $A_t$ denotes a time-varying level of aggregate productivity, and $N_{d,t}(j)$ denotes the amount of labor hired by the producer of variety $j$. Firms hire workers through competitive labor markets at wage rate $W_t$.

Firms sell their differentiated varieties domestically and to the rest of the world. We assume that prices are denominated in domestic currency, and that the law of one price holds at the firm-level: $P_{H,t}(j) = \xi_t P^*_{H,t}(j)$ for all $j \in [0,1]$, where $P_{H,t}(j)$ and $\xi_t P^*_{H,t}(j)$ denote firm $j$'s domestic currency price in the domestic market and the rest of the world, respectively, and $\xi_t$ is the nominal exchange rate (the price of the rest of the world's currency in terms of the domestic currency: that is, the value in domestic currency of one unit of foreign currency).

Moreover, we assume that firms set prices in a staggered fashion, as in Calvo (1983). That is, a measure $1-\theta$ of randomly selected firms sets new prices each period, with an individual firm's probability of re-optimizing in any given period being independent of the time elapsed since it last reset its price. Firms that cannot adjust their price in a given period commit to producing as much as demanded at the predetermined price. Firms are, thus, identical ex-ante, but are heterogeneous ex-post due to staggered pricing.

Finally, we assume that aggregate productivity follows an exogenous stochastic process given by $\log A_{t+1} = \rho_a \log A_t + \varepsilon_t^A$, where $\varepsilon_t^A$ is an iid shock with zero mean and standard deviation $\sigma_a$.

The problem solved by firm $j$ when setting a new price $\overline{P}_{H,t}$ in period $t$ consists of maximizing the current value of its dividend stream from the future states of the world in which it cannot adjust its price, subject to the production function and to the demand faced in each of the markets:

$$\max_{\overline{P}_{H,t}, N_{d,t+k}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left\{ \mathcal{M}_{t,t+k} \left[ Y_{t+k} \overline{P}_{H,t} - W_{t+k} N_{d,t+k} \right] \right\}$$

subject to

$$Y_{t+k} = \left( \frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \left( C_{H,t+k} + C^*_{H,t+k} \right) \quad \forall k$$

$$Y_{t+k} = A_{t+k} N_{d,t+k} \quad \forall k$$

where $\mathcal{M}_{t,t+k}$ denotes the representative household's stochastic discount factor between periods $t$ and $t+k$, and $P_{H,t}$ denotes the domestic good's price index.

Given complete markets, the stochastic discount factor is equal to the pricing kernel $\mathcal{M}_{t,t+k}(s^{t+k})$ for Arrow securities, which denotes the value of purchasing an Arrow security in state-$s^t$ that pays a unit of the good in state-$s^{t+k}$. This is how the representative household values future profit flows of the firms he owns.

### 3.3 Central bank

We study an economy in which the central bank sets the nominal interest rate $R_t$ of the one-period risk-free bond. Specifically, we assume that the monetary authority follows a standard Taylor rule, setting the nominal interest rate in response to deviations of output and CPI inflation from their steady-state values. As it is standard in the literature, we assume a certain degree of interest rate smoothing:

$$\log R_t = (1 - \rho_R) \log R_{ss} + \rho_R \log R_{t-1} + \phi_y \log \frac{Y_t}{Y_{ss}} + \phi_\Pi \log \frac{\Pi_t}{\Pi_{ss}}$$

where $\rho_R \in (0, 1)$ is the degree of interest rate smoothing, and $\Pi_t = \frac{P_t}{P_{t-1}}$ denotes CPI inflation.

### 3.4 Rest of the world

We assume that the domestic economy and the rest of the world feature symmetric initial conditions, such that there are zero net foreign asset holdings.

The domestic economy imports and exports varieties with the rest of the world, and we assume that the law of one price holds. Analogous to domestic exporters, we have that $P_{F,t}(j) = \xi_t P^*_{F,t}(j)$ for all $j \in [0, 1]$, where $P^*_{F,t}(j)$ is the price of the rest of the world's variety $j$ expressed in the producer's currency.

The rest of the world supplies and demands goods in a symmetric fashion to the domestic economy. That is, we assume that the demand for all goods produced by the domestic economy is given by $C^*_{H,t} = \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t$, where $\eta$ denotes the elasticity of substitution between domestic and foreign goods.

The representative household in the rest of the world trades a complete set of state-contingent Arrow securities with the domestic economy. Now, given that the small open economy is infinitesimaly small relative to the rest of the world, there is no distinction between the CPI and the domestic price level in the rest of the world. We thus have that $P^*_{F,t} = P^*_t$, which we normalize to unity. Similarly, given that the small open economy is infinitesimally small relative to the rest of the world, we do not solve the equilibrium for the rest of the world explicitly, but assume that aggregate output evolves according to the following autoregressive process:

$$\log Y_{t+1}^* = \rho_{y^*} \log Y_t^* + \varepsilon_t^{Y^*}$$

where $\varepsilon_t^{Y^*}$ is an iid shock with zero mean and standard deviation $\sigma_{y^*}$.

## 3.5 Equilibrium

A competitive equilibrium of this economy consists of policy functions $\{C_t, C_{H,t}, C_{F,t}, N_{d,t}, N_{s,t}, Y_t, C_t^*, \alpha_t\}_{t=0}^{\infty}$, exogenous variables $\{A_t, Y_t^*\}_{t=0}^{\infty}$, and prices $\{P_t, P_{F,t}, P_{H,t}, \Pi_t, \mathcal{M}_{t,t+1}, R_t, \xi_t, W_t\}_{t=0}^{\infty}$ such that the following conditions hold:

1. Given prices, policy functions solve the representative household's problem.
2. Given prices, policy functions solve the firms' problem.
3. Central bank sets $R_t$ following a Taylor rule.
4. Labor markets clear: $N_{s,t} = \int_0^1 N_{d,t}(j)dj \; \forall t$
5. Financial markets clear: $D_t^*(s^t) = 0 \; \forall s^t \forall t$, where $D_t^*(s^t)$ denotes holdings of period-$t$ state-$s^t$ Arrow securities by the rest of the world.
6. Domestic variety $j$'s market clears: $Y_t(j) = C_{H,t}(j) + C_{H,t}^*(j) \; \forall t$, where $C_{H,t}^*(j)$ denotes the consumption of the domestic variety $j$ in the rest of the world.

## 3.6 Additional definitions

Before we proceed with the analysis of the model, we now define objects that will be studied in subsequent sections. The terms of trade $\mathcal{S}_t$ are defined as $\mathcal{S}_t := \frac{P_{F,t}}{P_{H,t}}$; the real exchange rate $\mathcal{Q}_t$ is defined as $\mathcal{Q}_t := \frac{\xi_t P_t^*}{P_t}$; aggregate domestic output $Y_t$ is defined as $Y_t := \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$; and the trade balance relative to output $nx_t$ is defined as $nx_t := \left(\frac{1}{Y_{ss}}\right)\left(Y_t - \frac{P_t}{P_{H,t}} C_t\right)$, where $Y_{ss}$ denotes steady-state output, and the trade balance is expressed in terms of domestic output.

Finally, we define the output gap $Y_t^{gap}$ as $Y_t^{gap} := \frac{Y_t}{Y_t^{flex}}$, where $Y_t^{flex}$ consists of aggregate domestic output (as defined above) from a model with flexible prices (that is, with $\theta = 0$).

# 4 Time-varying trade wedge, demand for imports, and trade elasticities

In this section we complete our presentation of the model by specifying the functional form that we assume for the time-varying trade wedge $\alpha_t$. We then investigate its implications for the imports demand equation that we introduced in Section 2, as well as for the implied trade elasticities.

We assume that the imports trade wedge $\alpha_t$ in equation (2) is time-varying and evolves as a function of aggregate absorption ($C_t$ in the model), and the price of imports relative to absorption ($P_{F,t}/P_t$ in the model). Specifically, we assume that $\alpha_t$ takes the following functional form:

$$\alpha_t := \alpha + \psi_C \ln\left(\frac{C_t}{C_{ss}}\right) + \psi_{P_F} \ln\left(\frac{P_{F,t}/P_t}{P_{F,ss}/P_{ss}}\right),$$

where $\alpha \in [0,1]$ measures the degree of trade openness in the economy, while $\psi_C$ and $\psi_{P_F}$ are parameters that control the responsiveness of the time-varying trade wedge to deviations of aggregate absorption $C_t$ and the relative price of imports $P_{F,t}/P_t$ from their steady-state values.

While $\alpha_t$ is a function of endogenous variables, we assume that the household takes it as given when making decisions. Specifically, we assume that the household does not internalize the impact of his decisions on the equilibrium value of $\alpha_t$ (through their impact on $C_t$ and $P_{F,t}/P_t$).

Our specification of the trade wedge has three main features. First, if $\psi_C = \psi_{P_F} = 0$, then we have that $\alpha_t = \alpha$, as in standard trade models. Therefore, our model nests standard models of international trade and monetary policy, such as Gali and Monacelli (2005). Second, note that $\alpha_t = \alpha$ in the steady-state for any values of $\psi_C$ and $\psi_{P_F}$. Therefore, our model with a time-varying trade wedge and its counterpart with a constant trade wedge imply the same identical steady-state allocations and prices. This feature will ease the comparison of the business cycle dynamics implied by these two economies. Finally, note that, while alternative specifications with these properties exist, ours is motivated by its implications for the trade elasticities, as we show at the end of this section.

We now begin to examine the implications of this specification of $\alpha_t$ for the import demand equation implied by the model and its corresponding trade elasticities. In the following Lemma, we derive the demand for imports under the time-varying trade wedge assumed above.

**Lemma.** *With a time-varying trade wedge* $\alpha_t := \alpha + \psi_{P_F} \log \frac{P_{F,t}/P_t}{P_{F,ss}/P_{ss}} + \psi_C \log \frac{C_t}{C_{ss}}$, *the imports demand equation is given by* $\log C_{F,t} = \mu + (\psi_{P_F} - \eta) \log \frac{P_{F,t}}{P_t} + (1 + \psi_C) \log C_t$, *where* $\mu \in \mathbb{R}$ *is a function of structural parameters.*

This Lemma shows that our model implies a log-linear import demand equation that is a function of aggregate absorption and the price of imports relative to the price of absorption. Therefore, this equation is isomorphic to equation (1):

$$\log M_t = \kappa + \varphi \log \frac{p_{m,t}}{P_t} + \chi \log A_t + \nu_t.$$

These equations are identical in the particular case in which $\kappa = \mu$, $\varphi = \psi_{P_F} - \eta$, $\chi = 1 + \psi_C$, and $\nu_t = 0$. The following Proposition formalizes the implications of these findings for the implied trade elasticities.

**Proposition.** *With a time-varying trade wedge* $\alpha_t := \alpha + \psi_{P_F} \log \frac{P_{F,t}/P_t}{P_{F,ss}/P_{ss}} + \psi_C \log \frac{C_t}{C_{ss}}$*, the price elasticity of imports is* $\psi_{P_F} - \eta$*, and the income elasticity of imports is* $1 + \psi_C$*. In contrast, with a constant trade wedge* ($\psi_{P_F} = \psi_C = 0$)*, the price elasticity of imports is* $-\eta$*, and the income elasticity of imports is* $1$*.*

This proposition shows that the trade elasticities implied by our model are a one-to-one function of the time-varying trade wedge parameters $\psi_{P_F}$ and $\psi_C$. This property of $\alpha_t$ constitutes our main motivation for choosing this particular functional form for the time-varying trade wedge. In particular, by appropriately choosing the values of these parameters, our model can replicate the price and income elasticities of imports that we observe in the data. This is the approach that we pursue in the following section for calibrating these parameters.

# 5 Quantitative analysis

We now study the business cycle dynamics implied by the model and contrast them with those of a model with a constant trade wedge. We focus both on international trade variables and macroeconomic variables that are relevant for monetary policy. First, we examine the implications of the model for the volatility of these variables. We then investigate the channels that account for our findings by studying the response of our economy to a positive productivity shock. To simplify the analysis, throughout this section, we assume that the central bank follows a Taylor rule in which the interest rate only responds to deviations of inflation from its steady-state level; we relax this assumption in the next section.

## 5.1 Calibration

The model is calibrated to match salient features of the Canadian economy. The parameters of the trade wedge, which are specific to our framework, are calibrated based on the trade

| Parameter | Description | Value |
|---|---|---|
| $\beta$ | Discount factor | 0.99 |
| $\theta$ | Calvo parameter | 0.75 |
| $\phi$ | Elasticity of labor supply | 3 |
| $\rho$ | Coefficient of relative risk aversion | 1 |
| $\eta$ | Armington elasticity | 1.5 |
| $\sigma$ | Elasticity of substitution across varieties | 6 |
| $\rho_a$ | Persistence of domestic productivity shocks | 0.66 |
| $\rho_{y^*}$ | Persistence of foreign output shocks | 0.86 |
| $\sigma_a$ | Standard deviation of domestic productivity shocks | 0.0071 |
| $\sigma_{y^*}$ | Standard deviation of foreign output shocks | 0.0078 |
| $\rho_{a,y^*}$ | Correlation between domestic and foreign shocks | 0.3 |

Table 2: Parameterization, Preferences and Technology

elasticities that we estimate in Section 2. The rest of the parameters are standard, and we take their values from Gali and Monacelli (2005), who also calibrate them for Canada. The parameter values are reported in Tables 2 and 3.

A period in the model corresponds to a quarter in the data. Then, given our findings from the previous section, we set $\psi_C = 1.0659$ and $\psi_{P_F} = 1.5023$ which correspond to a price elasticty of 0.002 and an income elasticity of 2.066.

We choose $\beta = 0.99$, which corresponds to a riskless annual return of about 4% in steady-state. The share of firms that cannot adjust prices in a given period $\theta$ is set to 0.75, which implies that firms adjust prices every 4 quarters, on average. We set $\phi$ to 3, which corresponds to a labor supply elasticity of $\frac{1}{3}$. The elasticity of substitution across varieties is set to $\sigma = 6$, which implies that the steady-state mark-up is equal to 1.2. We choose $\alpha$, which in steady-state equals the imports to GDP ratio, to be 0.4. This value corresponds to its value for Canada. Finally, we set the coefficient of relative risk aversion $\rho$ to 1 and the Armington elasticity $\eta$ to 1.5, following Backus, Kehoe, and Kydland (1992).

The shock processes for aggregate productivity and output in the rest of the world are calibrated to match features of the data for Canada and the U.S., respectively, using the estimates of Gali and Monacelli (2005). Specifically, they fit AR(1) processes using data on labor productivity in Canada and the U.S., and estimate their degree of persistence and the

| Parameter | Description | Value |
|---|---|---|
| $\alpha_m$ | Degree of trade openness | 0.4 |
| $\psi_C$ | Consumption coefficient in trade wedge | 1.0659 |
| $\psi_{P_F}$ | Foreign price coefficient in trade wedge | 1.5023 |
| $\phi_m$ | Taylor rule weight on current inflation | 1.5 |
| $\phi_y$ | Taylor rule weight on output gap | 0 |
| $\rho_R$ | Interest rate smoothing | 0 |

Table 3: Parameterization, International Trade and Monetary Policy

volatility of the innovations.

Finally, in contrast to Gali and Monacelli (2005), we assume that the central bank follows a Taylor rule in which the interest rate only responds to deviations of inflation from its steady-state level. Following Taylor (1993), we set the coefficient on inflation to 1.5. In the next section, we relax this functional form and assume more general Taylor-type monetary rules.

For each of the models that we study, we compute 50 simulations of 200 periods. In doing so, we compute the simulations for each of the models based on the same realization of the shocks.

## 5.2 International trade fluctuations

We begin by examining the implications of our model for business cycle fluctuations of international trade variables, which we contrast both the constant trade wedge model and the data. In particular, we investigate the extent to which our model can account for salient features of international trade fluctuations that are not targeted directly in our calibration.

The results are reported in Table 4. In the first row, we report moments of the data for Canada computed byEngel and Wang (2011). In the next two rows we report the moments implied by each of the models. We find that our model improves significantly over the constant trade wedge model. In particular, in our model, net exports become counter-cyclical as observed in the data. Moreover, exports are now more volatile than GDP whereas, in the constant trade wedge model, they are roughly as volatile. Finally, imports are now positively correlated with both GDP and exports, whereas in the constant trade wedge model these correlations are, respectively, low and negative.

Therefore, by targeting explicitly the price and income elasticities estimated from the

| | Standard deviation | | | Correlation with GDP | | | |
|---|---|---|---|---|---|---|---|
| | NX/GDP | M | X | NX/GDP | M | X | corr(M,X) |
| Data | 0.66 | 3.15 | 2.65 | -0.12 | 0.74 | 0.66 | 0.62 |
| Constant trade wedge | 0.47 | 2.34 | 1.10 | 0.25 | 0.18 | 0.97 | -0.05 |
| Time-varying trade wedge | 0.72 | 3.66 | 2.05 | -0.77 | 0.69 | 0.96 | 0.47 |

Note: Standard deviation of imports and exports is relative to GDP.

Table 4: International Trade Fluctuations

| | Standard deviation | | | | | |
|---|---|---|---|---|---|---|
| | $Y$ | $\Pi$ | $R$ | $Q$ | Output gap | $C$ |
| Constant trade wedge | 0.75% | 0.27% | 0.40% | 0.65% | 0.53 | 1.26 |
| Time-varying trade wedge | 0.56% | 0.30% | 0.45% | 0.73% | 0.68 | 1.77 |

Note: Standard deviation of output gap and consumption are relative to GDP.

Table 5: Business Cycle Fluctuations, Moments

data, our model is able to better account for key features of international trade variables not targeted in the calibration. In particular, the root-mean-squared error between our model and the data is 47.1% lower than for the standard model.

### 5.3 Business cycle fluctuations

Having shown that our model can better account for international trade fluctuations, we now examine its implications for the business cycle dynamics of variables that are relevant for monetary policy. The results are reported in Table 5, in which we compare both models.

We find that there are significant differences in the business cycle dynamics implied by the two models. In particular, we find that our model implies a higher volatility of inflation and the real exchange rate, while featuring a lower volatility of output. Even though the central bank conducts monetary policy based on the same Taylor rule in the two models, the higher volatility of inflation in our model leads to a higher volatility of the nominal interest rate.

Moreover, we also find that the volatility of the output gap and consumption are higher, relative to GDP, suggesting that accounting for international trade fluctuations may have significant welfare implications when designing monetary policy.

Therefore, in a model that captures salient features of international trade fluctuations, the central bank may want to react to aggregate fluctuations differently than in the standard model. We explore this further in Section 6 where we compute optimal monetary policy under the two specifications of our model.

## 5.4 Impulse response functions

To understand the channels that account for the different business cycle dynamics implied by the two models, we now study the response of the economy to a one-standard-deviation positive domestic productivity shock (Figures 1 and 2).[6]

We find qualitative and quantitative differences in the response of economic variables across the two models. In the constant trade wedge model, a positive productivity shock in the domestic economy leads to an increase in consumption. Given complete markets, these gains are shared with the rest of the world through a depreciation of the real exchange rate, making imports more expensive. This leads to a decrease in imports despite the increase in aggregate expenditures. Therefore, the increase of aggregate consumption is driven by the increase of domestic consumption.

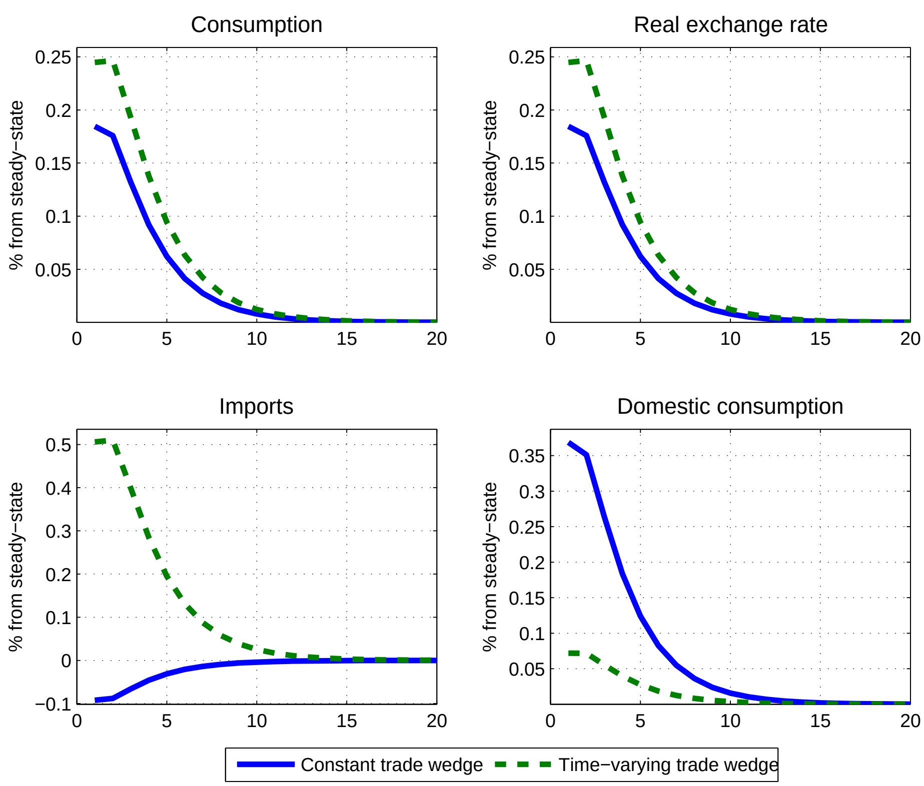


Figure 1: Impulse reponse functions to a positive productivity shock

[6]We report the impulse response functions corresponding to a one-standard-deviation positive foreign output shock in the Appendix.

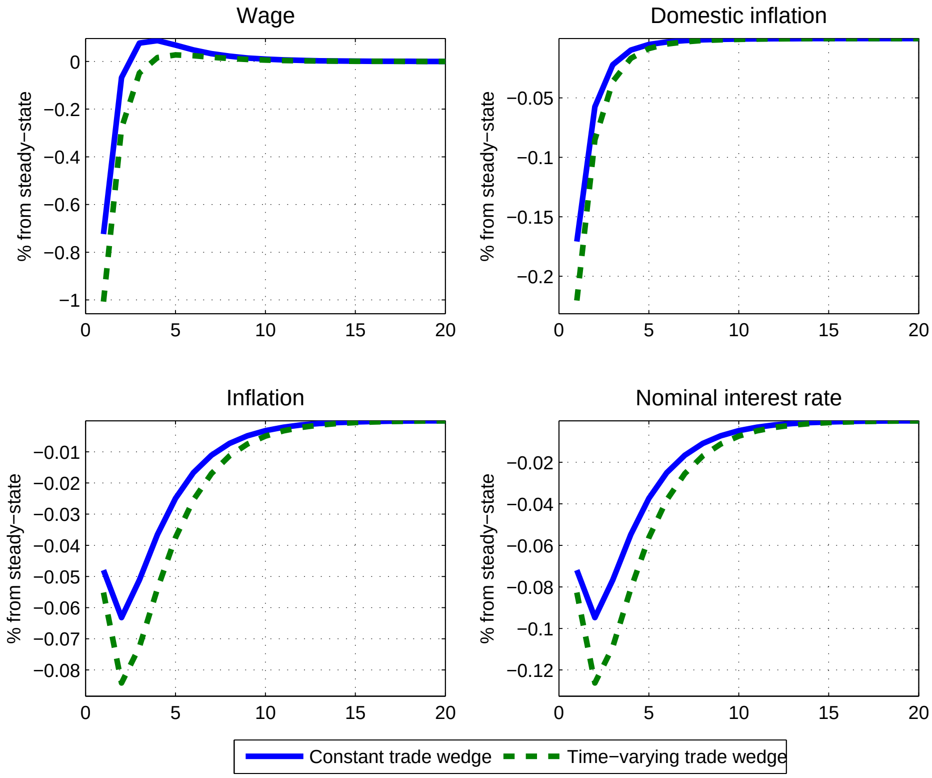


Figure 2: Impulse reponse functions to a positive productivity shock

In contrast, in the time-varying trade wedge model, imports increase despite a larger increase of consumption and, therefore, a larger depreciation of the real exchange rate. These differences are driven by the lower price elasticity and the higher income elasticity of imports featured by our model. On the one hand, the low price elasticity reduces the response of imports to the depreciation of the real exchange rate. On the other hand, the higher income elasticity increases the response of imports to the increase in aggregate consumption. The result is an increase of imports, in contrast to the constant trade wedge model. Aggregate consumption is then driven by an increase of both domestic consumption and imports.

These differences have implications for variables that are relevant for monetary policy. In particular, we focus on the dynamics for aggregate inflation. In the constant trade wedge model, wages decrease, leading to lower inflation. With sticky prices, the effect of a positive productivity shock on wages is ambiguous. On the one hand, firms that cannot adjust prices decrease their demand for labor since they can produce more output per unit of labor but cannot decrease their prices to take advantage of their higher productivity. On the other hand, those firms that can adjust prices choose lower prices, increase the demand for their goods and, therefore, their demand for labor.

The net impact on wages depends on the price stickiness parameter $\theta$ and the elasticity of substitution $\sigma$. In our parameterization, the former effect dominates. As a result, both domestic and aggregate inflation decrease. Given the Taylor rule, the central bank responds to the decrease in inflation by lower the real interest rate.

In the time-varying trade wedge model we find a larger decrease in wages and, therefore, inflation. The lower increase of domestic consumption decreases the demand faced by firms, therefore decreasing the amount of labor that they hire. This leads to a larger decrease of wages and, thus, domestic and aggregate inflation. This larger response of inflation then leads to a larger decrease of the nominal interest rate.

Our findings show that the different reaction of imports featured by our model changes the quantitative response of variables relevant for monetary policy. This suggests that the optimal response of a central bank to aggregate fluctuations may be different in the two models. We explore this further in the next section.

# 6 Optimal monetary policy

In this section we study the optimal design of monetary policy in an economy that can account for international trade fluctuations, and contrast the results with those of a constant trade wedge model.

To do so, we compute the Ramsey problem of a central bank that is constrained to follow a Taylor rule. Specifically, the central bank chooses the Taylor rule coefficients that maximize the lifetime expected utility of the consumer in a competitive equilibrium. In contrast to the previous section, we now consider a Taylor rule in which the nominal interest rate responds smoothly to deviation of inflation and output from their steady-state levels.

We contrast the optimal policies across the two models, and then quantify their implications for business cycle dynamics.

## 6.1 Constrained Ramsey problem

To study the optimal design of monetary policy, we consider a central bank that chooses coefficients $\rho_R$, $\phi_y$, and $\phi_\Pi$ of the Taylor rule

$$\log R_t = (1-\rho_R)\log R_{ss} + \rho_R \log R_{t-1} + \phi_y \log \frac{Y_t}{Y_{ss}} + \phi_\Pi \log \frac{\Pi_t}{\Pi_{ss}}$$

to maximize the lifetime expected utility of the representative household in a competitive equilibrium.

Formally, the problem is given by

$$\max_{\rho_R,\phi_Y,\phi_\Pi} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left[C_t^{CE}(\rho_R,\phi_Y,\phi_\Pi), N_t^{CE}(\rho_R,\phi_Y,\phi_\Pi)\right]$$

subject to the Taylor rule and the competitive equilibrium allocations, where $C_t^{CE}(\rho_R, \phi_Y, \phi_\Pi)$ and $N_t^{CE}(\rho_R, \phi_Y, \phi_\Pi)$ are the consumption and labor allocations, respectively, corresponding to a competitive equilibrium in which the central bank follows a Taylor rule with coefficients $\rho_R$, $\phi_Y$, and $\phi_\Pi$.

## 6.2 Optimal Taylor rule

We begin by contrasting the Taylor rule coefficients that maximize welfare under each of the models. The optimal Taylor rule coefficients that result from the constrained Ramsey problem are reported in Table 6.

Qualitatively, in both models, the optimal response of monetary policy to aggregate fluctuations is the same. The nominal interest rate should decrease when output is above its steady-state (to reduce the output gap) or when inflation is below its steady-state. In addition, in both models the planner finds it optimal to smooth nominal interest rate adjustments.

Quantitatively, however, we find significant differences in the optimal response to aggregate fluctuations across the two models. In the time-varying trade wedge model the optimal degree of interest rate smoothing is smaller than in the standard model, which implies that the central bank is more impatient. Therefore, the Ramsey planner reacts more strongly to short-run deviations of inflation and output from their steady-state levels.

Our findings suggest that accounting for international trade fluctuations is important for the design of monetary policy.

| | $\rho$ | $\phi_Y$ | $\phi_\Pi$ |
|---|---|---|---|
| Constant trade wedge | 0.91 | -0.04 | 0.28 |
| Time-varying trade wedge | 0.67 | -0.13 | 0.92 |

Table 6: Optimal Taylor rule

## 6.3 Quantitative implications

We now evaluate the extent to which differences in the optimal Taylor rule coefficients implied by the two models matter for business cycle dynamics.

We begin by assuming that the true data generating process is the time-varying trade wedge model. We use this model to contrast the business cycle dynamics implied by two alternative monetary policy rules.

| | Standard deviation (%) | | |
|---|---|---|---|
| | Inflation | Output gap | $R$ |
| Optimal policy | 0.12 | 0.55 | 0.20 |
| Suboptimal policy | 0.21 | 0.66 | 0.11 |

Table 7: Quantitative implications

First, we consider a central bank that follows the optimal Taylor rule corresponding to the time-varying trade wedge model, as in the second row of Table 6.

Second, we consider a central bank that follows the optimal Taylor rule corresponding to the contant trade wedge model, as in the first row of Table 6. This case captures a central bank that behaves sub-optimally by choosing the optimal Taylor rule coefficients based on the wrong data generating process (i.e. the constant trade wedge model).

We compare the implied moments for variables relevant to monetary policy: inflation, the output gap, and the nominal interest rate. The results are reported in Table 7.

We find that, when the central bank carries out the sub-optimal policy corresponding to the constant trade wedge model, both inflation and the output gap would be substantially more volatile. In particular, inflation would be almost twice as volatile whereas the output gap would be 20% more volatile.

As observed in Table 6, the optimal policy corresponding to the constant trade wedge model implies a lower response to inflation and output gap deviations, leading to higher volatility of these variables when this policy is followed under the true data generating process (i.e. the time-varying trade wedge model).

# 7 Conclusion

In this paper, we study the role of trade openness for the design of monetary policy. We extend a standard small open economy model of monetary policy to capture cyclical fluctuations of international trade flows, and parametrize it to match key features of the data. We find that accounting for trade fluctuations matters for monetary policy. Specifically, we find that the volatility of variables relevant to the design of monetary policy are higher when the central bank follows the optimal policy based on a model that cannot account for international trade fluctuations.

By showing that trade fluctuations matter for the design of monetary policy, these results put in context previous findings in the literature. In particular, in a model that better accounts for international trade fluctuations, without significantly affecting other business cycle dynamics, we find that central banks should conduct monetary policy differently than implied by standard models. Our paper, thus, introduces recent developments from the literature on international trade dynamics to the established literature that studies monetary policy in open economies.

# References


Alessandria, G., J. Kaboski, and V. Midrigan (2013): "Trade wedges, inventories, and international business cycles," *Journal of Monetary Economics*, 60(1), 1–20.

Alessandria, G., J. P. Kaboski, and V. Midrigan (2010): "The Great Trade Collapse of 2008–09: An Inventory Adjustment&quest," *IMF Economic Review*, 58(2), 254–294.

Anderson, J. E., and E. van Wincoop (2003): "Gravity with Gravitas: A Solution to the Border Puzzle," *American Economic Review*, 93(1), 170–192.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1992): "International real business cycles," *Journal of political Economy*, pp. 745–775.

Calvo, G. A. (1983): "Staggered prices in a utility-maximizing framework," *Journal of monetary Economics*, 12(3), 383–398.

Chor, D., and K. Manova (2012): "Off the cliff and back? Credit conditions and international trade during the global financial crisis," *Journal of International Economics*, 87(1), 117–133.

De Paoli, B. (2009): "Monetary policy and welfare in a small open economy," *Journal of International Economics*, 77(1), 11–22.

Diewert, W. E. (1978): "Superlative index numbers and consistency in aggregation," *Econometrica: Journal of the Econometric Society*, pp. 883–900.

Eaton, J., and S. Kortum (2002): "Technology, geography, and trade," *Econometrica*, pp. 1741–1779.

Eaton, J., S. Kortum, B. Neiman, and J. Romalis (2011): "Trade and the global recession," Discussion paper, National Bureau of Economic Research.

Ehemann, C., A. J. Katz, and B. R. Moulton (2002): "The chain-additivity issue and the US national economic accounts," *Journal of Economic and Social Measurement*, 28(1), 37–49.

Engel, C., and J. Wang (2011): "International trade in durable goods: Understanding volatility, cyclicality, and elasticities," *Journal of International Economics*, 83(1), 37–52.

Feenstra, R. C. (1994): "New product varieties and the measurement of international prices," *The American Economic Review*, pp. 157–177.

Fisher, I. (1922): *The making of index numbers: a study of their varieties, tests, and reliability*, no. 1. Houghton Mifflin.

Gali, J., and T. Monacelli (2005): "Monetary policy and exchange rate volatility in a small open economy," *The Review of Economic Studies*, 72(3), 707–734.

Heathcote, J., and F. Perri (2002): "Financial autarky and international business cycles," *Journal of Monetary Economics*, 49(3), 601–627.

Houthakker, H. S., and S. P. Magee (1969): "Income and price elasticities in world trade," *The Review of Economics and Statistics*, pp. 111–125.

Krugman, P. (1980): "Scale economies, product differentiation, and the pattern of trade," *The American Economic Review*, pp. 950–959.

Leibovici, F., and M. E. Waugh (2014): "International Trade and Intertemporal Substitution," Discussion paper, National Bureau of Economic Research.

Levchenko, A. A., L. T. Lewis, and L. L. Tesar (2010): "The collapse of international trade during the 2008–09 crisis: in search of the smoking gun," *IMF Economic Review*, 58(2), 214–253.

Lombardo, G., and F. Ravenna (2014): "Openness and optimal monetary policy," *Journal of International Economics*, 93(1), 153–172.

Marquez, J. (2013): *Estimating trade elasticities*, vol. 39. Springer Science & Business Media.

Melitz, M. J. (2003): "The impact of trade on intra-industry reallocations and aggregate industry productivity," *Econometrica*, 71(6), 1695–1725.

Taylor, J. B. (1993): "Discretion versus policy rules in practice," in *Carnegie-Rochester conference series on public policy*, vol. 39, pp. 195–214. Elsevier.

Whelan, K. (2002): "A guide to US chain aggregated NIPA data," *Review of income and wealth*, 48(2), 217–233.

# Appendix: Impulse response to foreign output shock

This section completes the analysis of Section 5.4 by presenting the impulse response functions implied by our model to a one-standard-deviation foreign output shock, and contrasting them with those implied by the constant trade wedge model. The dynamics of these economies in response to such shock are illustrated in Figures 3 and 4.

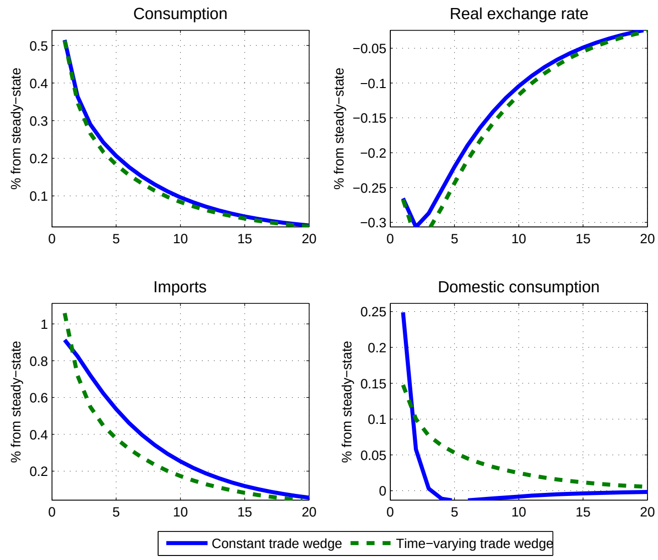


Figure 3: Impulse reponse functions to a positive foreign output shock

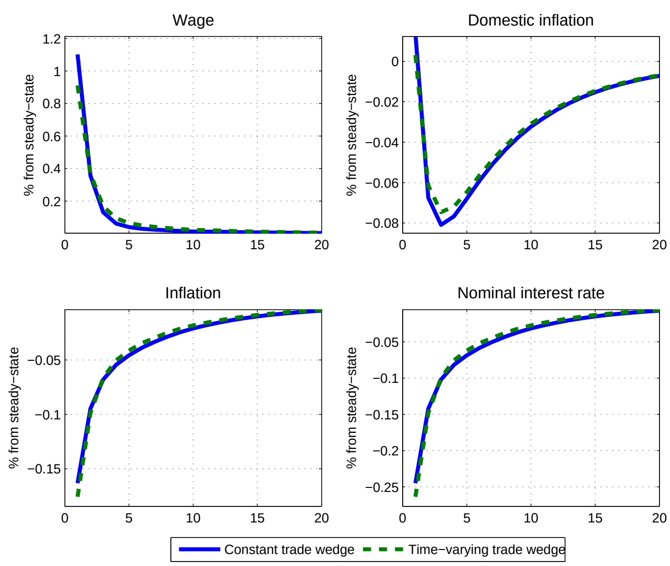


Figure 4: Impulse reponse functions to a positive foreign output shock